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CALCULATION OF ELECTRON DENSITY IN THE VICINITY OF A BLUNT BODY WITHIN THE FRAMEWORK OF VARIOUS MODELS OF DIFFUSION IN HYPERSONIC FLOW OVER IT

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The question of the influence of the choice of the diffusion model on the distribution of flow parameters in the problem of the flow of a hypersonic air stream over a blunt body is discussed on the basis of a numerical solution of the Navier–Stokes equation.

The high-temperature air in the region of the shock layer between the surface of a body and the bow shock wave (SW) consists of a complicated, multicomponent, partially ionized gas mixture. The solution of the problem of the flow of such mixture over a body within the framework of the complete system of Navier–Stokes equations lies at the limit of the possibilities of modern computers. The question of the degree of complexity of the model which must be used to describe the multicomponent medium is very important from this point of view. In problems of hypersonic air flow over bodies one can construct a hierarchy of models, starting with the most complicated, in which one takes into account 11 components of the mixture reacting with each other (N_2, O_2, NO, N, O, NO^+, N_2^+, O_2^+, N^+, O^+, e), the nonequilibrium of the internal degrees of freedom, and processes of multicomponent diffusion, viscosity, and heat conduction for a sufficiently large number of approximations in Sonine polynomials for the coefficients of transfer of the charged components. The next simpler model has seven components (N_2, O_2, NO, O, N, NO^+, e), in it the internal degrees of freedom are in equilibrium, and transfer processes are taken into account within the framework of the complete system of Navier–Stokes equations. There can be further simplifications of the model, connected with discarding individual terms in the Navier–Stokes equations, as a result of which the type of system changes, with dividing the entire region of flow into subregions, in each of which simpler equations are used (Euler equations, boundary-layer equations), etc. And simplifications are also possible within the framework of any model.

Besides the comparison of the results of the solution with experiment, the comparison with data obtained on the basis of a more complicated model can serve as a criterion for the correctness of the adopted assumptions. A solution has now been obtained within the framework of the seven-component model for the problem of air flow over a blunt body. In the velocity range of 4-6 km/sec at the pressure and densities corresponding to altitudes of 70-100 km above sea level the seven-component model, in which the leading ionization process is associative ionization $N + O \rightleftarrows e + NO^+$, describes the properties of the medium sufficiently well. This model is still complicated for making mass calculations of flow over bodies, however, since each variant of the calculations consumes large amounts of computer time. Below we analyze the possibility of simplifications of the seven-component model of air through approximate allowance for the diffusional properties of the mixture. We consider the question of how the accuracy in assigning the cross sections of elastic collisions of particles of the gas mixture influences the distribution of concentrations of the charged components, and we also investigate the difference arising in the case when multicomponent diffusion is replaced by binary diffusion. In addition, the correctness of the standard assumption that the medium is quasi-neutral is analyzed on the basis of a calculation of the induced electric fields and the space charge in the vicinity of the body. Concrete results

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are given for the following conditions: flow over a body with spherical blunting of radius 13 cm, velocity of the oncoming stream $u_\infty = 5.5$ km/sec, Mach number M = 20, Reynolds number Re = 300 and 600. The gas mixture consists of seven components: 1) O_2; 2) O; 3) N; 4) NO; 5) NO^+; 6) e; 7) N_2. To describe the diffusion processes in such a mixture we use equations obtained in a 10-moment approximation by Grad's method with coefficients of multicomponent diffusion calculated with an accuracy corresponding to allowance for the first Sonine polynomial. It is assumed that the medium is quasi-neutral, external electric fields are absent, the electric current equals zero everywhere in the region of flow, and the electron temperature equals the temperature of the heavy particles. Under these assumptions we can eliminate the strength of the self-consistent electric field from the diffusion equations. Neglecting processes of baro- and thermodiffusion, we can write the equations describing the density distributions of components of the mixture in the vicinity of the body in the form [1]

$$\frac{\partial \rho_i}{\partial t} + \operatorname{div} \rho_i \mathbf{u} + \operatorname{div} \mathbf{J}_i = \omega_i, \ \rho_i = m_i n_i; \tag{1}$$

$$\sum_{j=1}^{5} a^*_{ij} \mathbf{J}_j = \nabla \rho_i, \quad i = 1, \ldots, 5. \tag{2}$$

Here m_i, n_i, and $\mathbf{J}_i$ are the mass of a particle, the number density, and the diffusional mass flux of the i-th component of the mixture; $\mathbf{u}$ is the average-mass velocity of the mixture; ω_i is the rate of change of the density of the i-th component due to chemical reactions; elements of a matrix inverse to the matrix constructed from the coefficients a^*_{ij} yield the coefficients of multicomponent diffusion in the ambipolar approximation. The expressions for a^*_{ij} have the form

$$\begin{gathered} a^*_{ij} = a_{ij} - a_{i7}, \quad i, j = 1, \ldots, 4, \\ a^*_{i5} = a_{i5} - a_{i7} + \frac{m_6}{m_5}(a_{i6} - a_{i7}), \quad i = 1, \ldots, 4, \\ a^*_{5j} = 0{,}5\,(a_{5j} - a_{57} + a_{6j} - a_{67}), \quad j = 1, \ldots, 4, \\ a^*_{55} = 0{,}5\left[a_{55} - a_{57} + a_{65} - a_{67} + \frac{m_6}{m_5}(a_{56} - a_{57} + a_{66} - a_{67})\right], \\ a_{ij} = \frac{m_i}{(m_i + m_j)\,\tau_{ji} kT}, \ i \neq j, \ \frac{1}{\tau_{ij}} = \frac{16}{3} n_j \left(\frac{kT}{2\pi\mu_{ij}}\right)^{1/2} Q_{ij}, \\ a_{ii} = -\sum_{\substack{j=1 \\ j \neq i}}^{7} \frac{m_j}{(m_i + m_j)\,\tau_{ij} kT}, \ \mu_{ij} = \frac{m_i m_j}{m_i + m_j}; \end{gathered} \tag{3}$$

$$Q_{ij} = 2\pi \int v^5 e^{-v^2} (1 - \cos\chi_{ij})\, b\,db\,dv, \tag{4}$$

where Q_{ij} is a transport cross section for elastic collisions of particles; T is the temperature; k is the Boltzmann constant; the numbering of the components is given above. For elastic collisions between charged particles we use Coulomb scattering cross sections cut off at the Debye distance,

$$Q_{ij} = \frac{\pi e^4}{2(kT)^2} \ln \Lambda_{ij}, \ \Lambda_{ij} = \frac{(kT)^{3/2}}{(4\pi n_5)^{1/2} e^3} \tag{5}$$

(e is the proton charge). For the remaining cross sections of interactions (besides electron–neutral-particle cross sections) we used data calculated by the method of [2]. Expressions for the integrals (4) giving their dependence on temperature in the range of 200-25,000°K can be represented as

$$\lg Q^*_{ij} = \sum_k c_k (\lg T)^k, \ Q_{ij} = 10^{16} Q^*_{ij} \ \text{cm}^2. \tag{6}$$

Here the temperature T is expressed in degrees Kelvin; coefficients c_k of the polynomials for the interaction of the NO^+ ion with neutral particles are given in Table 1.

In calculations of the cross sections Q_{ij} we used interaction potentials with allowance for the polarization attraction of an ion and a neutral particle, important at low temperatures, as well as the dispersion interaction and repulsion in the region of high collision energies. The latter types of interaction were written in a form analogous to that for the interaction of the corresponding neutral particles.

For the cross sections of collisions of neutral particles we used the data of Table 2 [2].

TABLE 1

c_i	O_2-NO^+	$O-NO^+$	$N-NO^+$	$NO-NO^+$	N_2-NO^+
c_0	−1.28940	5.82752	5.39566	−1.62362	6.27833
c_1	6.17367	−2.82542	−2.26721	6.56184	−3.05815
c_2	−3.49261	0.590961	0.400264	−3.65424	0.655657
c_3	0.762411	−0.0448437	−0.025474	0.793381	−0.0511894
c_4	−0.0589641	0	0	−0.061387	0

For the cross sections of collisions between electrons and particles of the neutral components of this mixture there are mainly the experimental results of [3, 4], on the basis of which we used the cross sections given in Table 3 in the calculations ($Q_{6j} = 10^{16}Q^*_{6j}$ cm^2).

As mentioned earlier, the system of diffusion equations (2) is written in the approximation of 10 moments in a solution of the kinetic equations by the Maxwell−Grad method of moments. By solving this system for the diffusional fluxes $\mathbf{J}_i$, we can express $\mathbf{J}_i$ through the density gradients of the components, in which the coefficients of multicomponent diffusion will be calculated in the approximation corresponding to the use of one Sonine polynomial (the first approximation for the transfer coefficients in the Chapman−Enskog method). Under the conditions being considered, the gas mixture is weakly ionized and the interaction between charged particles plays no significant role, so that the accuracies of the 10-moment approximation for the diffusion coefficients are sufficient [1], especially if we consider that the data for the cross sections of the interaction of electrons with neutral particles are very approximate.

The diffusion fluxes determined from the system (2) are used in the equations (1) of continuity of the components, which also include the quantities ω_i describing the rates of formation of the components in chemial reactions. In the seven-component model under consideration we allow for seven reactions: dissociation of O_2, N_2, and NO molecules, three reactions of formation of nitric oxide NO, and the reaction of associative ionization N + O $\rightleftarrows$ NO^+ + e. The rate constants of these reactions were taken from [5]. Equations (1) and (2) were solved jointly with the equations of continuity and of conservation of momentum and energy for the mixture as a whole, written in the Navier−Stokes approximation [6]. In solving the problem of hypersonic flow over a body with spherical blunting we took the surface temperature as given, assuming that the surface of the body is chemically neutral relative to the components of the mixture. We also calculated variants of ideal catalyticity of the wall relative to the charged components.

Below we give the distributions of dimensionless electron density γ_e along the axis of symmetry of the flow and along a ray drawn opposite to the stream from the center of the sphere at an angle of 40° to the axis of symmetry. Since data on the cross sections of collisions between charged particles (NO^+ ions and electrons) with neutral ones are very approximate, it is interesting to estimate the influence of this factor on the results of calculations of γ_e ($\gamma_e = \rho_6\mu_\infty/\rho\mu_6$, ρ is the density of the mixture, μ_∞ is the molecular weight of the mixture in the oncoming stream, and μ_6 is the molecular weight of the electron component). For this purpose we made calculations with collision cross sections differing from those given above. In these variants we assumed that the cross sections of collisions between ions and neutral particles are constant and do not depend on temperature. The values of the cross sections were taken in accordance with Table 4.

For the cross sections Q_{ea} we took the values $Q_{eN_2} = 10^{-15}$ cm^2 and $Q_{e\text{-}NO} = 7\cdot10^{-16}$ cm^2; the remaining values of Q_{ea} are the same as in Table 3. The values given in Table 4 correspond approximately to those of [3, 7]. We note that the cross sections (6) from Table 1 vary rather strongly with variation of the temperature. Thus, the cross section $Q_{NO^+\text{-}N_2}$ increases almost twofold with a decrease in temperature from $1.1\cdot10^4$°K (the temperature behind the shock wave at M = 20) to $2\cdot10^3$°K.

Variants of the calculation with data on the collision cross sections given in Tables 1-3 are designated as MV below for brevity in the presentation. Variants of the calculation with constant cross sections of collisions between charged and neutral particles (see Table 4) will be called MC. In the case of constant cross sections, the coefficients a^*_{ij} in Eqs. (2) vary in proportion to $T^{-1/2}$.

TABLE 2

c_i	O_2–O	O_2–N	O_2–NO	O_2–N_2	O–N	O–NO	O–N_2	N–NO	N–N_2	NO–N_2
c_0	1.687036	2.36535	4.09548	1.885781	1.642237	2.41046	1.952193	2.16046	2.3171	2.16334
c_1	0.11281	−0.406347	−2.05342	−0.0450509	0.109589	−0.432299	−0.0548706	−0.268072	−0.318046	−0.229716
c_2	−0.07395	0.0205946	0.543926	−0.0319548	−0.0663888	0.0262529	−0.0466622	0	0	0
c_3	0	0	−0.05145	0	0	0	0	0	0	0

TABLE 3

Q^*_{6j}				
e–O	e–N	e–N_2	e–O_2	e–NO
4,5	5	13	5	10

TABLE 4

$Q_{ij}\cdot 10^{16}$, cm^2				
O_2–NO^+	O–NO^+	N–NO^+	NO–NO^+	N_2–NO^+
30	30	30	50	30

Calculations were also made from a considerably simpler diffusion model, when it is assumed that the diffusion flux of the i-th component is proportional to the density gradient of this component (model of binary diffusion). Here the coefficient of ambipolar diffusion is taken as the binary diffusion coefficient of ions, since it is assumed that the medium is quasi-neutral and an electric current is absent in the entire flow region. In addition, in this model it was assumed that the ratios of the binary diffusion coefficients of the components to the viscosity of the mixture are constant. Calculation variants within the framework of the model of binary diffusion are designated as BD. The rate constants of the chemical reactions were taken as the same in all the variants.

Let us examine the results of the calculations. In Fig. 1 we show the distributions of the dimensionless gas-dynamic parameters of the mixture (the density ρ^*, the pressure p^*, and the temperature T^* – curves 1-3) along the axis of flow symmetry (solid lines) and along a ray drawn from the center of spherical blunting at an angle of 40° to the axis of symmetry (dashed lines) for M = 20 and Re = 300, where $\rho^* = \rho/\rho_\infty$, $p^* = p/\rho_\infty u_\infty^2$, $T^* = c_{p\infty}T/u_\infty^2$, and the index ∞ marks values of the parameters in the oncoming stream. The quantity y, representing the distance from the surface of the body along a normal, normalized to the radius of the sphere, is laid out along the abscissa axis here and in the other figures.

It is seen that the SW structure is located in the region of $0.15 \lesssim y \lesssim 0.25$ along the critical line and in the region of $0.2 \lesssim y \lesssim 0.36$ along the 40° ray, and on the critical streamline the SW width is ~0.13, which comprises almost half of the compressed shock layer (corresponding to $0 \lesssim y \lesssim 0.25$). With greater distance from the axis of symmetry the intensity of the SW decreases owing to its curvature. On the 40° ray the pressure and density behind the SW start to fall due to expansion of the stream.

In Fig. 2 we give the distributions of dimensionless electron density along the same rays (1 is the critical streamline and 2 is the 40° ray) and variants of calculations within the framework of different models. The values of the concentrations of the components in the oncoming stream are taken in accordance with [8]. It is seen that there are pronounced differences in the character of the γ_e profiles between the models of multicomponent and binary diffusion. In the case of multicomponent diffusion the γ_e gradients are considerably smaller in the entire region of the compressed shock layer. On the critical line in the MV and MC variants the electron density starts to increase in the region of $y \lesssim 0.34$ ahead of the SW front (which corresponds to $y \approx 0.25$) due to processes of multicomponent diffusion. For the MV variant the maximum value of $\gamma_e = 9.9\cdot 10^{-8}$ in the compressed shock layer is 1.4 times lower than in the calculation based on the BD model ($1.35\cdot 10^{-7}$). The γ_e profiles in MV and MC variants are similar, but there is a quantitative difference, reaching about 25% in the region of the maximum values of γ_e.

The results are similar for M = 20 and Re = 600 (Fig. 3, notation same as in Fig. 2). The difference in the maximum electron density in the compressed layer is ~12% between the MV and BD variants and ~7% between the MV and MC variants. Flatter γ_e profiles are obtained within the framework of the model of multicomponent diffusion, both in the region of the SW structure and in the region of the compressed layer near the body, where the electron density

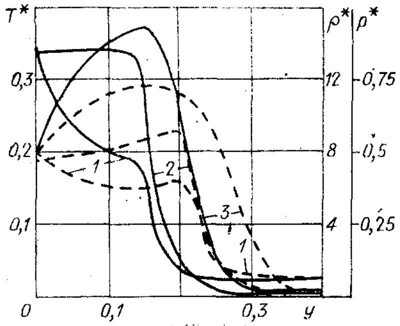

Fig. 1

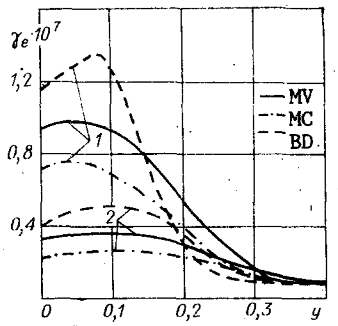

Fig. 2

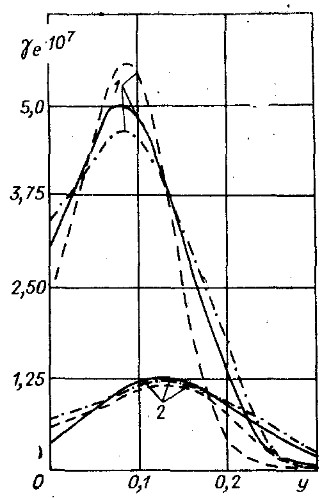

Fig. 3

falls due to recombination processes. We also note that multicomponent diffusion promotes the upstream escape of electrons behind the SW front.

Calculations of the distributions of concentrations of the components, made in the case of a perfectly catalytic wall for charged particles, show that the influence of the boundary conditions on the γ_e profile is felt only in a narrow layer near the surface of the body.

From these data we can conclude that in problems where the density distribution of electrons in the vicinity of the body is important, the simplified BD model can be used only for estimates of γ_e. Varying the cross sections of elastic collisions between charged and neutral particles within the framework of the same model of multicomponent diffusion does not alter the qualitative character of the behavior of the γ_e profiles. Here the quantitative difference lies in the range of 10-20% for a rather significant difference in the cross sections. Thus, in the MC variant the Q_{N_2}-NO^+ cross section is about 1.5 times larger than the characteristic value of this cross section in the MV variant.

Let us consider the question of the accuracy of the approximation of quasi-neutrality in problems of hypersonic flow over a body in the presence of ionization processes in the shock layer in its vicinity. The initial diffusion equations [1] contain terms with the electric field strength $\mathbf{E}$. Using one of them, e.g., for the ions, we write

$$\mathbf{E} = \frac{kT}{e\rho_5}\left(\nabla\rho_5 - \sum_{j=1}^{7} a_{5j}\mathbf{J}_j\right). \qquad (7)$$

The coefficients a_{5j} are given by Eqs. (3). The quantities appearing on the right side of (7) are calculated in the process of solving the problem of hypersonic flow over the body. Substituting E from (7) into the Poisson equation

$$\operatorname{div}\mathbf{E} = 4\pi q, \; q = e(n_5 - n_6), \qquad (8)$$

we can find the distribution of the electric space charge q in the vicinity of the body in the first approximation with respect to the small parameter equal to the ratio of the square of the characteristic Debye distance to the square of the characteristic size of the body. We note that in the zeroth approximation with respect to this parameter q = 0, and the equality $n_5 = n_6$ is used instead of Eq. (8) to solve the problem of hypersonic flow over the body.

In Fig. 4 we show the distribution of the dimensionless components of the electric field strength along rays inclined to the axis of symmetry at angles of 10° (solid lines) and 30° (dashed lines) for the conditions M = 20 and Re = 300 in the oncoming stream and the MV variant of description of the diffusion properties. Curves 1 give the profiles of the field component E_y^* along the normal to the surface of the body and curves 2 give the profiles of the field component E_x^* in the direction perpendicular to the normal. The field components are normlaized to $E_0 = ku_\infty^2/c\rho_\infty eL$, where L is the radius of spherical blunting of the body. Under these conditions E_0 = 0.19 V/cm. The results are given for a perfectly catalytic surface for ions and electrons. It is seen from the calculations that the maximum electric field strength is reached near the surface of the body and has the order of several volts per centimeter. The normal field component E_y^* changes sign in the region of the SW structure.

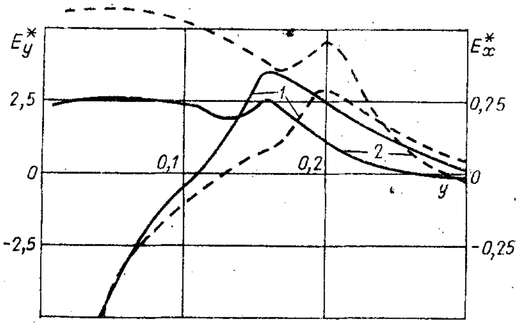

Fig. 4

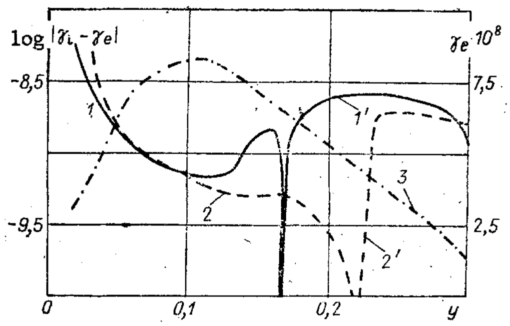

Fig. 5

In Fig. 5 we give the distribution on a logarithmic scale of the quantity $\gamma_i - \gamma_e$, which is proportional to the density of the electric space charge q (lines 1 and 1' are along the 10° ray and lines 2 and 2' are along the 30° ray), where $\gamma_i = \rho_5 \mu_\infty / \rho \mu_5$ (μ_5 is the molecular weight of the ions). In the region of the SW the space charge changes sign: Curves 1 and 2 correspond to $q > 0$ and curves 1' and 2' to $q < 2'$ to $q < 0$. Line 3 gives the distribution of the quasineutral charged-particle density γ_e. It is seen that the maximum value of the ratio $|\gamma_i - \gamma_e|/\gamma_e$ does not exceed 0.1. Thus, the assumption that the medium flowing over the body is quasineutral is satisfied in the range of conditions under consideration with an accuracy corresponding to the acuracy of the adopted model for describing the diffusion processes. In fact, as was noted earlier, the difference in the γ_e profiles between the MV and MC variants also lies within limits of 10-20% and is due to the uncertainty in the data on the cross sections of collisions between charged and neutral particles.

By integrating this density distribution of space charge q over the volume (over the entire calculated region from the axis of symmetry to the extreme ray and then over angles from 0 to 2π), one can find the total electric charge formed in the calculated region in the vicinity of the body during hypersonic flow over it. The stream flowing onto the body is electrically neutral (it is natural to assume that a space charge is absent from the oncoming stream), and it is also assumed that in steady flow the electric currents equal zero in the entire flow field. Therefore, by virtue of the law of conservation of charge, a charge of the opposite sign must form on the body over which the flow occurs: A body moving in air at a hypersonic velocity becomes electrified. We note that the above method of calculating the electrification of bodies in hypersonic flow enables one to find the electric charge of the body to within the charge that forms in the thin double layer near the surface of the body.

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